Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced Level

Friday 30 January 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation or integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions on this paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. $$f(x) = 2x^3 - 8x^2 + 7x - 3.$$

Given that $x = 3$ is a solution of the equation $f(x) = 0$, solve $f(x) = 0$ completely.

(5)

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2. (*a*) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n}(6r^2 + 4r - 1) = n(n + 2)(2n + 1).$$

(5)

(*b*) Hence, or otherwise, find the value of $\sum_{r=11}^{20}(6r^2 + 4r - 1)$.

(2)

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3. The rectangular hyperbola, H, has parametric equations $x = 5t$, $y = \frac{5}{t}$, $t \neq 0$.

(*a*) Write the cartesian equation of H in the form $xy = c^2$.

(1)

Points A and B on the hyperbola have parameters $t = 1$ and $t = 5$ respectively.

(*b*) Find the coordinates of the mid-point of AB.

(3)

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4. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n}\frac{1}{r(r+1)} = \frac{n}{n+1}.$$

(5)

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5. $$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20.$$

(*a*) Show that the equation $f(x) = 0$ has a root α in the interval $[1.1, 1.2]$.

(2)

(*b*) Find $f'(x)$.

(3)

(*c*) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures.

(4)

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6. A series of positive integers u_1, u_2, u_3, ... is defined by

$$u_1 = 6 \text{ and } u_{n+1} = 6u_n - 5, \text{ for } n \geq 1.$$

Prove by induction that $u_n = 5 \times 6^{n-1} + 1$, for $n \geq 1$.

(5)

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7. Given that $\mathbf{X} = \begin{pmatrix} 2 & a \\ -1 & -1 \end{pmatrix}$, where a is a constant, and $a \neq 2$,

(*a*) find $\mathbf{X}^{-1}$ in terms of a.

(3)

Given that $\mathbf{X} + \mathbf{X}^{-1} = \mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix,

(*b*) find the value of a.

(3)

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8. A parabola has equation $y^2 = 4ax$, $a > 0$. The point Q $(aq^2, 2aq)$ lies on the parabola.

(*a*) Show that an equation of the tangent to the parabola at Q is

$$yq = x + aq^2.$$

(4)

This tangent meets the y-axis at the point R.

(*b*) Find an equation of the line l which passes through R and is perpendicular to the tangent at Q.

(3)

(*c*) Show that l passes through the focus of the parabola.

(1)

(*d*) Find the coordinates of the point where l meets the directrix of the parabola.

(2)

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9. Given that $z_1 = 3 + 2\text{i}$ and $z_2 = \dfrac{12 - 5\text{i}}{z_1}$,

(*a*) find z_2 in the form $a + \text{i}b$, where a and b are real.

(2)

(*b*) Show, on an Argand diagram, the point P representing z_1 and the point Q representing z_2.

(2)

(*c*) Given that O is the origin, show that $\angle POQ = \dfrac{\pi}{2}$.

(2)

The circle passing through the points O, P and Q has centre C. Find

(*d*) the complex number represented by C,

(2)

(*e*) the exact value of the radius of the circle.

(2)

10.
$$\mathbf{A} = \begin{pmatrix} 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the transformations described by each of the matrices **A**, **B** and **C**.

(4)

It is given that the matrix **D** = **CA**, and that the matrix **E** = **DB**.

(*b*) Find **D**.

(2)

(*c*) Show that $\mathbf{E} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$.

(1)

The triangle *ORS* has vertices at the points with coordinates (0, 0), (–15, 15) and (4, 21). This triangle is transformed onto the triangle *OR'S'* by the transformation described by **E**.

(*d*) Find the coordinates of the vertices of triangle *OR'S'*.

(4)

(*e*) Find the area of triangle *OR'S'* and deduce the area of triangle *ORS*.

(3)

TOTAL FOR PAPER: 75 MARKS

END

January 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
1	$x-3$ is a factor	B1
	$\text{f}(x) = (x-3)(2x^2-2x+1)$	M1 A1
	Attempt to solve quadratic i.e. $x = \dfrac{2 \pm \sqrt{4-8}}{4}$	M1
	$x = \dfrac{1 \pm \text{i}}{2}$	A1 **[5]**

Notes:

First and last terms in second bracket required for first M1
Use of correct quadratic formula for their equation for second M1

Question Number	Scheme	Marks
2 (a)	$6\sum r^2+4\sum r-\sum 1=6\frac{n}{6}(n+1)(2n+1)+4\frac{n}{2}(n+1),-n$	M1 A1, B1
	$=\frac{n}{6}(12n^2+18n+6+12n+12-6)$ or $n(n+1)(2n+1)+(2n+1)n$	M1
	$=\frac{n}{6}(12n^2+30n+12)=n(2n^2+5n+2) = n(n+2)(2n+1)$ *	A1 (5)
(b)	$\sum_{r=1}^{20}(6r^2+4r-1)$ - $\sum_{r=1}^{10}(6r^2+4r-1) = 20\times22\times41-10\times12\times21$	M1
	$= 15520$	A1 (2) **[7]**

Notes:

(a) First M1 for first 2 terms, B1 for $-n$
Second M1 for attempt to expand and gather terms.
Final A1 for correct solution only

(b) Require (r from 1 to 20) subtract (r from 1 to 10) and attempt to substitute for M1

Question Number	Scheme	Marks
3 (a)	$xy = 25 = 5^2$ or $c = \pm 5$	B1 (1)
(b)	A has co-ords (5, 5) and B has co-ords (25, 1)	B1
	Mid point is at (15, 3)	M1A1 (3) **[4]**

Notes:

(a) $xy = 25$ only B1, $c^2 = 25$ only B1, $c = 5$ only B1

(b) Both coordinates required for B1
Add theirs and divide by 2 on both for M1

Question Number	Scheme	Marks
4	When n = 1, LHS = $\frac{1}{1\times 2}=\frac{1}{2}$, RHS = $\frac{1}{1+1}=\frac{1}{2}$. So LHS = RHS and result true for n = 1	B1
	Assume true for $n = k$; $\sum_{r=1}^{k}\frac{1}{r(r+1)}=\frac{k}{k+1}$ and so $\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	M1
	$\sum_{r=1}^{k+1}\frac{1}{r(r+1)}=\frac{k(k+2)+1}{(k+1)(k+2)}=\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)^2}{(k+1)(k+2)}=\frac{k+1}{k+2}$	M1 A1
	and so result is true for $n = k + 1$ (and by induction true for $n \in \mathbf{Z}^+$)	B1 **[5]**

Notes:

Evaluate both sides for first B1
Final two terms on second line for first M1
Attempt to find common denominator for second M1.
Second M1 dependent upon first.
$\frac{k+1}{k+2}$ for A1
'Assume true for $n = k$ 'and 'so result true for $n = k+1$' and correct solution for final B1

Question Number	Scheme	Marks
5 (a)	attempt evaluation of f(1.1) and f(1.2) (– looking for sign change)	M1
	f(1.1) = 0.30875 , f(1.2) = -0.28199 Change of sign in f(x) $\Rightarrow$ root in the interval	A1 (2)
(b)	$\mathrm{f}'(x)=\frac{3}{2}x^{-\frac{1}{2}}-9x^{-1\frac{1}{2}}$	M1 A1 A1 (3)
(c)	f (1.1) = 0.30875.. f′ (1.1) = -6.37086…	B1 B1
	$x_1=1.1-\frac{0.30875...}{-6.37086..}$	M1
	= 1.15(to 3 sig.figs.)	A1 (4) **[9]**

Notes:

(a) awrt 0.3 and -0.3 and indication of sign change for first A1
(b) Multiply by power and subtract 1 from power for evidence of differentiation and award of first M1
(c) awrt 0.309 B1and awrt -6.37 B1 if answer incorrect
Evidence of Newton-Raphson for M1
Evidence of Newton-Raphson and awrt 1.15 award 4/4

Question Number	Scheme	Marks
6	At $n=1$, $u_n = 5\times 6^0 + 1 = 6$ and so result true for $n = 1$	B1
	Assume true for $n = k$; $u_k = 5\times 6^{k-1} + 1$,and so $u_{k+1} = 6(5\times 6^{k-1} + 1) - 5$	M1, A1
	$\therefore u_{k+1} = 5\times 6^k + 6 - 5 \quad \therefore u_{k+1} = 5\times 6^k + 1$	A1
	and so result is true for $n = k + 1$ and by induction true for $n \geq 1$	B1 **[5]**

Notes:
6 and so result true for $n = 1$ award B1
Sub u_k into u_{k+1} or M1 and A1 for correct expression on right hand of line 2
Second A1 for $\therefore u_{k+1} = 5\times 6^k + 1$
'Assume true for $n = k$' and 'so result is true for $n = k + 1$' and correct solution for final B1

Question Number	Scheme	Marks
7 (a)	The determinant is a - 2	M1
	$\mathbf{X}^{-1} = \frac{1}{a-2}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$	M1 A1 (3)
(b)	$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1
	Attempt to solve $2 - \frac{1}{a-2} = 1$, or $a - \frac{a}{a-2} = 0$, or $-1 + \frac{1}{a-2} = 0$, or $-1 + \frac{2}{a-2} = 1$	M1
	To obtain a =3 only	A1 cso (3) **[6]**
	Alternatives for (b) If they use $\mathbf{X^2 + I = X}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1 If they use $\mathbf{X^2 + X^{-1} = O}$, they can score the B1then marks for solving If they use $\mathbf{X}^3 + \mathbf{I} = \mathbf{O}$ they need to identify **I** for B1, then attempt to solve suitable equation for M1 and obtain a = 3 for A1	

Notes:

(a) Attempt *ad-bc* for first M1

$\frac{1}{\det}\begin{pmatrix} -1 & -a \\ 1 & 2 \end{pmatrix}$ for second M1

(b) Final A1 for correct solution only

Question Number	Scheme	Marks
8 (a)	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 4a$	M1
	The gradient of the tangent is $\frac{1}{q}$	A1
	The equation of the tangent is $y - 2aq = \frac{1}{q}(x - aq^2)$	M1
	So $yq = x + aq^2$ *	A1 (4)
(b)	R has coordinates (0, aq)	B1
	The line l has equation $y - aq = -qx$	M1A1 (3)
(c)	When $y = 0$ $x = a$ (so line l passes through (a, 0) the focus of the parabola.)	B1 (1)
(d)	Line l meets the directrix when $x = -a$: Then $y = 2aq$. So coordinates are ($-a$, $2aq$)	M1:A1 (2) **[10]**

Notes:

(a) $\frac{dy}{dx} = \frac{2a}{2aq}$ OK for M1

Use of $y = mx + c$ to find c OK for second M1

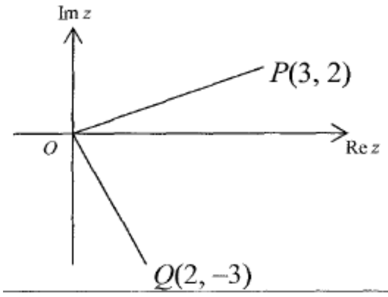
Correct solution only for final A1

(b) -1/(their gradient in part a) in equation OK for M1

(c) They must attempt $y = 0$ or $x = a$ to show correct coordinates of R for B1

(d) Substitute $x = -a$ for M1.

Both coordinates correct for A1.

Question Number	Scheme	Marks
9 (a)	$z_2 = \frac{12-5\text{i}}{3+2\text{i}} \times \frac{3-2\text{i}}{3-2\text{i}} = \frac{36-24\text{i}-15\text{i}-10}{13}$	M1
	$= 2-3\text{i}$	A1 (2)
(b)	 P: B1, Q: B1ft	B1, B1ft (2)
(c)	grad. OP × grad. $OQ = \frac{2}{3} \times -\frac{3}{2}$	
	$= -1 \quad \Rightarrow \angle POQ = \frac{\pi}{2} \quad (\ast)$	
OR	$\angle POX = \tan^{-1}\frac{2}{3}$, $\angle QOX = \tan^{-1}\frac{3}{2}$	
	$\text{Tan}(\angle POQ) = \frac{\frac{2}{3}+\frac{3}{2}}{1-\frac{2}{3}\times\frac{3}{2}}$ M1	M1
	$\Rightarrow \angle POQ = \frac{\pi}{2} \quad (\ast)$ A1	A1 (2)
(d)	$z = \frac{3+2}{2} + \frac{2+(-3)}{2}\text{i}$	M1
	$= \frac{5}{2} - \frac{1}{2}\text{i}$	A1 (2)
(e)	$r = \sqrt{\left(\frac{5}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$	M1
	$= \frac{\sqrt{26}}{2}$ or exact equivalent	A1 (2) **[10]**

Notes:

(a) $\times \frac{3-2i}{3-2i}$ for M1

(b) Position of points not clear award B1B0

(c) Use of calculator / decimals award M1A0

(d) Final answer must be in complex form for A1

(e) Radius or diameter for M1

Question Number	Scheme	Marks
10 (a)	**A** represents an enlargement scale factor $3\sqrt{2}$ (centre O)	M1 A1
	B represents reflection in the line $y = x$	B1
	C represents a rotation of $\frac{\pi}{4}$, i.e.45° (anticlockwise) (about O)	B1 (4)
(b)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} 3 & -3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} -3 & 3 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 0 & -15 & 4 \\ 0 & 15 & 21 \end{pmatrix} = \begin{pmatrix} 0 & 90 & 51 \\ 0 & 0 & 75 \end{pmatrix}$ so (0, 0), (90 , 0) and (51 , 75)	M1A1A1A1 (4)
(e)	Area of $\Delta\, OR'S'$ is $\frac{1}{2} \times 90 \times 75 = 3375$	B1
	Determinant of **E** is –18 or use area scale factor of enlargement So area of $\Delta\, ORS$ is $3375 \div 18 = 187.5$	M1A1 (3) **[14]**

Notes:

(a) Enlargement for M1
$3\sqrt{2}$ for A1

(b) Answer incorrect, require **CD** for M1

(c) Answer given so require **DB** as shown for B1

(d) Coordinates as shown or written as $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 90 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 51 \\ 75 \end{pmatrix}$ for each A1

(e) 3375 B1
Divide by theirs for M1